

Unit 2 - Solving Equations

AFTER COMPLETION OF THIS UNIT, YOU WILL BE ABLE TO...

- SOLVE ONE, TWO, AND MULTI-STEP EQUATIONS (VARIABLES ON BOTH SIDES)
- CREATE AND SOLVE AN EQUATION FROM A CONTEXT
- SOLVE A LITERAL EQUATION (MULTIPLE VARIABLES) FOR A SPECIFIED VARIABLE

TIMELINE FOR UNIT 2

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
16 TH DAY 1 — SOLVING 1 & 2 STEP EQUATIONS	17 TH DAY 1 — SOLVING 1 & 2 STEP EQUATIONS	18 TH DAY 2 — MULTI-STEP EQUATIONS	19 TH DAY 2 — MULTI-STEP EQUATIONS	20 TH DAY 2 — MULTI-STEP EQUATIONS
23 RD EQUATION PRACTICE	24 TH DAY 3 — CREATING AND SOLVING EQUATIONS FROM A CONTEXT	25 TH DAY 3 — CREATING AND SOLVING EQUATIONS FROM A CONTEXT	26 TH DAY 4 — CREATING EQUATIONS WITH DISTRIBUTION	27 TH DAY 5 — CONSECUTIVE NUMBER EQUATIONS
30 TH EQUATIONS FROM A CONTEXT PRACTICE	31 ST DAY 6 — LITERAL EQUATIONS	1 ST DAY 6 — LITERAL EQUATIONS	2 ND DAY 6/7 LITERAL EQUATIONS	3 RD LITERAL EQUATION REVIEW
6 TH REVIEW DAY	7 TH UNIT 2 ASSESSMENT			

Day 1 – Solving One & Two Step Equations - Notes

Remember, an **expression** is a mathematical “phrase” composed of terms, coefficients, and variables that stands for a single number, such as $3x + 1$ or $x^2 - 1$. We use Properties of Operations to simplify algebraic expressions. Expressions do NOT contain equal signs.

An Algebra Expression does NOT have an = sign.

$$4n^2 + 7$$

An “Equation” does have an Equals sign.

$$4n^2 + 7 = 11$$

An **equation** is a mathematical “sentence” that says two expressions are equal to each other such as $3x + 1 = 5$. We use Properties of Equality (inverse operations) to solve algebraic equations. Equations contain equal signs.

When solving equations, you must perform **inverse operations**, which means you have to perform the operation opposite of what you see. You must also remember the operation you perform on one side of the equation must be performed to the other side.

Informal		Formal		
Operation	Inverse	Property	General Example	Example 1
Addition		Addition Property of Equality	If $a = b$, then $a + c = b + c$	If $x - 4 = 8$, then $x = 12$
Subtraction		Subtraction Property of Equality	If $a = b$, then $a - c = b - c$	If $x + 5 = 7$, then $x = 2$
Multiplication		Multiplication Property of Equality	If $a = b$, then $ac = bc$	If $\frac{x}{2} = 9$, then $x = 18$
Division		Division Property of Equality	If $a = b$, then $\frac{a}{c} = \frac{b}{c}$	If $2x = 10$, then $x = 5$

No More “Cancelling”

When you first learned to solve equations in middle school, you might have used the words “cancel”. We are no longer going to use the word “cancel”. Take a look at the following examples:

$$x - 120 = 80$$

$$\begin{array}{r} +120 \quad +120 \\ x \quad \quad = 200 \end{array} \quad \leftarrow \begin{array}{l} \text{Adding the opposite} \\ \text{Additive inverse} \\ \text{Adding to zero} \end{array}$$

$$\frac{k}{2} = 16$$

$$\begin{array}{r} \cancel{k} \quad \cancel{\times 2} \\ k = 32 \quad \checkmark \end{array} \quad \leftarrow \begin{array}{l} \text{Multiplying by the Reciprocal} \\ \text{Multiplicative Inverse} \\ \text{Divides/Multiplies to one} \end{array}$$

Additive Inverse	A number plus its inverse equals 0.	$a + -a = 0$	$7 + -7 = 0$
Multiplicative Inverse (Reciprocal)	A number times its reciprocal equals 1.	$a \cdot \frac{1}{a} = 1$	$3 \cdot \frac{1}{3} = 1$

Solving One Step Equations

Practice: Solve each equation.

1. $x - 4 = 3$ Operation You See: _____ Inverse Operation: _____

2. $y + 4 = 3$ Operation You See: _____ Inverse Operation: _____

3. $\frac{s}{3} = 9$ Operation You See: _____ Inverse Operation: _____

4. $6p = 12$ Operation You See: _____ Inverse Operation: _____

Practice: Solve each equation on your own.

a. $x - 6 = 10$

b. $-5d = 25$

c. $8 + m = -4$

d. $\frac{x}{7} = 1$

e. $y - (-9) = 2$

f. $\frac{1}{3}x = 6$

Solving Two Step Equations

When solving equations with more than one step, you still want to think about how you can “undo” the operations you see. For the following equations, describe the operations being performed on each variable (go in PEMDAS order). Then describe the inverses using a table.

a. $3x + 5 = 14$

b. $2n - 6 = 4$

c. $\frac{x-2}{4} = 1$

Practice: Solve each equation, showing all steps, for each variable.

1. $3x - 4 = 14$

2. $2x + 4 = 10$

3. $7 - 3y = 22$

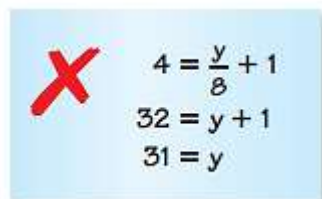
4. $0.5m - 1 = 8$

5. $-6 + \frac{x}{4} = -5$

6. $\frac{x-8}{4} = -5$

Error Analysis with Solving Equations

1. William solved the following equation on his homework last night. However, he solved it incorrectly. Describe the mistake William made and what he should have done instead. Then re-solve the equation to find the correct answer.



$$4 = \frac{y}{8} + 1$$

$$32 = y + 1$$

$$31 = y$$

Mistake: _____

Corrected Solution:

2. Tyler solved the following equation on his homework last night. However, he solved it incorrectly. Describe the mistake Tyler made and what he should have done instead. Then re-solve the equation to find the correct answer.



$$28y + 7 = 21$$

$$28y = 28$$

$$y = 1$$

Mistake: _____

Corrected Solution:

Day 1 – Solving One & Two Step Equations - Practice

Directions: Solve each equation, showing all work.

1. $m - 9 = -13$

2. $\frac{b}{3} = -6$

3. $-15x = 0$

4. $4x + 7 = 31$

5. $\frac{n}{5} - 4 = -2$

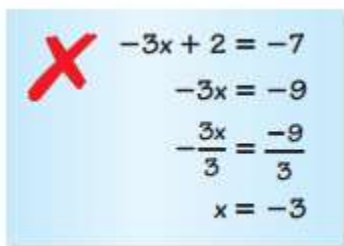
6. $\frac{y+4}{7} = -3$

7. $7x + 18 = -24$

8. $5 - 2x = 15$

9. $\frac{-7x - (-3)}{3} = 15$

10. Describe and correct the error:



$-3x + 2 = -7$
 $-3x = -9$
 $\frac{-3x}{3} = \frac{-9}{3}$
 $x = -3$

Day 2 – Solving Multi-Step Equations – Notes

Multi-step equations mean you might have to add, subtract, multiply, or divide all in one problem to isolate the variable. When solving multi-step equations, you are still using inverse operations, which is like doing PEMDAS in reverse order. However, your left and right sides should be completely simplified (combined like terms and distributed) BEFORE using inverse operations. Make it your goal to have a simplified equation that looks like the equations below before using inverse operations.

Variable + constant = variable + constant	$2x + 4 = x - 3$
Variable + constant = constant	$2x + 4 = -3$
Variable + constant = variable	$2x + 4 = x$

Multi - Step Equations with Combining Like Terms

Practice: Solve each equation, showing all steps, for each variable.

a. $-5n + 6n + 15 - 3n = -3$

b. $3x + 12x - 20 = 25$

c. $-2x + 4x - 12 = 40$

d. $14 = -3 + 7n - 4$

e. $6 + 5x + 6x = 17$

f. $12 = 2x - 8x$

g. $7x - 3 + 8 + 2x = 14$

h. $-4 - 6x - 4x = 6$

i. $7x - 4 - 1 - 2x = 15 + 5$

Multi - Step Equations with the Distributive Property

Practice: Solve each equation, showing all steps, for each variable.

a. $2(n + 5) = -2$

b. $4(2x - 7) + 5 = -39$

c. $6x - (3x + 8) = 16$

d. $6 + 4(4 + 2x) = 86$

e. $-8(x - 7) = 120$

f. $-6(x - 7) - 8x = 98$

g. $8(6x + 7) - 8(5 + 4x) = -32$

h. $-4(4 + 5x) - 5(-5x + 1) = -31$

i. $-3(-2x + 6) + 8(3x - 4) = -20$

Multi – Step Equations with Variables on Both Sides

Practice: Solve each equation, showing all steps, for each variable

a. $5p - 14 = 8p + 4$

b. $8x - 1 = 23 - 4x$

c. $5x + 34 = -2(1 - 7x)$

d. $-7(1 + 6x) = -7 + 8n$

e. $-39 - 6x = -5(x + 7) - 5$

f. $-5(2x - 1) + 5 = -4 - 8x$

g. $-6(5 + 2x) = 5(1 - x)$

h. $2(6 - 3x) = 2 - 2(4x + 8)$

i. $-2 + 5(1 + 3x) = 2x - 5(2x + 4)$

 Error Analysis with Solving Equations

1. Rachel solved the following equation on her homework. However, she solved it incorrectly. Describe the mistake Rachel made and what she should have done instead. Then resolve the equation to find the correct answer.

$$\begin{aligned}
 2(x + 3) &= -3(-x + 1) \\
 2x + 6 &= 3x - 3 \\
 5x + 6 &= -3 \\
 5x &= -9 \\
 x &= -\frac{5}{9}
 \end{aligned}$$

Mistake: _____

Correction Solution:

2. Mikayla solved the following equation on her homework. However, she solved it incorrectly. Describe the mistake Mikayla made and what she should have done instead. Then resolve the equation to find the correct answer.

$$\begin{aligned}
 -2(7 - y) + 4 &= -4 \\
 -14 - 2y + 4 &= -4 \\
 -10 - 2y &= -4 \\
 -2y &= 6 \\
 y &= -3
 \end{aligned}$$

Mistake: _____

Correction Solution:

Day 2 – Solving Multi-Step Equations – Practice

Directions: Solve each equation, showing all work.

1. $6(x + 1) + 5 = 35$

2. $-14 = -2(5 - x) + 16$

3. $\frac{4(x-8)}{5} = 16$

4. $-4x - 46 = 5x + 71$

5. $-4 - 5x = -4x + 9$

6. $4x + 31 = -67 - 3x$

7. $8x - 27 - 10 - 6x = 15$

8. $8(1 + 5x) + 5 = 13 + 5x$

9. $5(4x - 2) + 9 = 2(8x + 7)$

10. $7 - 2(-5x - 9) + 2x = 4(2x - 1) + 3(x + 12)$

Day 3 – Creating Equations from a Context – Notes

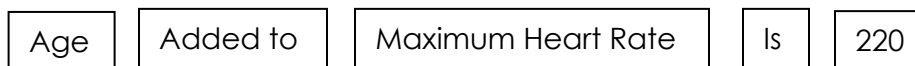
Earlier in our unit, you learned to write expressions involving mathematical operations. You used the following table to help you decode those written expressions. We are going to use those same key words along with words that indicate an expression will become part of an equation or inequality.

Addition	Subtraction	Multiplication	Division	Equals
Sum	Difference	Of	Quotient	Is
Increased by	Decreased by	Product	Ratio of	Equals
More than	Minus	Times	Percent	Will be
Combined	Less	Multiplied by	Fraction of	Gives
Together	Less than	Double	Out of	Yields
Total of	Fewer than	Twice	Per	Costs
Added to	Withdraws	Triple	Divided by	
Gained				
Raised				
Plus				

When taking a word problem and translating it to an equation or inequality, it is important to “**talk to the text**” or underline/highlight key phrases or words. By doing this it helps you see what is occurring in the problem.

Modeling Mathematics with Equations

A person’s maximum heart rate is the highest rate, in beats per minute, that the person’s heart should reach. One method to estimate the maximum heart rate states your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15 year old.



In the equation above, we did not know one of the quantities. When we do not know one of the quantities, we use a **variable** to represent the unknown quantity. When creating equations, it is important that whatever variable you use to represent the unknown quantity, you define or state what the variable represents.

Practice Examples: In the examples below, “talk to the text” as you translate your word problems into equations. Define a variable to represent an unknown quantity, create your equation, and then solve your equation.

1. Six less than four times a number is 18. What is the number?

Variables: _____

Equation: _____

2. You and three friends divide the proceeds of a garage sale equally. The garage sale earned \$412. How much money did each friend receive?

Variables: _____

Equation: _____

3. On her iPod, Mia has rock songs and dance songs. She currently has 14 rock songs. She has 48 songs in all. How many dance songs does she have?

Variables: _____

Equation: _____

4. Brianna has saved \$600 to buy a new TV. If the TV she wants costs \$1800 and she saves \$20 a week, how many months will it take her to buy the TV (4 weeks = 1 month)?

Variables: _____

Equation: _____

5. Mrs. Jackson earned a \$500 bonus for signing a one year contract to work as a nurse. Her salary is \$22 per hour. If her first week's check including the bonus is \$1204, how many hours did Mrs. Jackson work?

Variables: _____

Equation: _____

6. Morgan subscribes to a website for processing her digital pictures. The subscription is \$5.95 per month and 4 by 6 inch prints are \$0.19. How many prints did Morgan purchase if the charge for January was \$15.83?

Variables: _____

Equation: _____

Day 3 – Creating Equations from a Context – Practice

Write an equation that models the situation. You do NOT have to solve!

1. Five times the sum of e and 4 is equal to -7 .

2. Jamie buys 9 CDs at same price per CD and a cassette tape for \$9.45. His total bill was \$118.89.

Define a variable for each problem below. Then write an equation that can be used to model the following problem. Finally, use your equation to SOLVE the problem.

3. At a concert, Nabila purchased three t-shirts and a \$15 concert program. In total, Nabila spent \$90. Find the cost of a single t-shirt if they all had the same price.

Variables: _____

Model: _____

4. Oberon Cell Phone Company advertises service for 3 cents per minute plus a monthly fee of \$29.95. If Parker's phone bill for October was \$38.95, find the number of minutes he used.

Variables: _____

Model: _____

5. Jacqueline had \$20 to spend on 7 raffle tickets. After purchasing them she had \$6 left. How much did each raffle ticket cost?

Variables: _____

Model: _____

Day 4 - Creating Equations with Distribution – Notes

1. The daycare center charges \$120 for one week of care. Families with multiple children pay \$95 for each additional child per week. Write an equation for the total cost for one week of care in terms of the number of children. How many children does a family have if they spend \$405 a week in childcare?

Variables: _____

Equation: _____

2. The party store has a special on greeting cards. It charges \$14 for 4 greeting cards and \$1.50 for each additional card. Write an equation for the total cost of greeting cards in terms of the number of cards. What is the total cost for 9 greeting cards?

Variables: _____

Equation: _____

3. Clara has a coupon for \$10 off her favorite clothing store. The coupon is applied before any discounts are taken. The store is having a sale and offering 15% off everything. If Clara has \$50 to spend, how much can her purchases total before applying the discount to her coupon?

Variables: _____

Equation: _____

Day 4 - Creating Equations with Distribution – Practice

Directions: Create an equation to represent each scenario.

1. Naomi is deciding which online photo sharing service she wants to subscribe to. One of her options, Photo World, charges an initial membership fee of \$35 for the first month and then \$4 for each additional month.

2. James is going to join a gym which charges him \$89 for the first month and then \$26 for each additional month.

3. Josh is planning an anniversary dinner for his parents. At Luigi's, they charge him \$245 for the first 10 guests and then \$17.50 for each additional guest.

4. The Beach Shack rents boats for \$60 for the first three hours and \$30 for each hour after that.

5. Admission for groups of students to Ocean World is \$330 for the first twenty students. Each student after that must pay \$11.50.

6. Melanie runs a lawn mowing business and charges \$50 for the first time they mow your yard and \$25 for each additional time they come back to mow your lawn.

7. The zoo offers special admission rates for large groups of visitors. The zoo charges \$7.50 admission for the first visitor and \$5.50 for each additional visitor in the group. Write an equation for the total cost of admission in terms of the number of visitors. How much is admission for a group of 8 visitors?

8. The jewelry store has a special on shirts. If you purchase 2 shirts for \$65, each additional shirt is \$24.99. Write an equation that represents that total cost of shirts based on the number of shirts purchased. What is the total cost of purchasing 4 shirts?

Day 5 – Consecutive Number Equations – Notes

1. Consider the following numbers:

45, 46, 47

102, 103, 104

30, 31, 32

99, 100, 101

- What patterns do you notice?
- How does the second number compare to the first number?
- How does the third number compare to the first number?

2. Consider the following numbers:

32, 34, 36

98, 100, 102

50, 52, 54

78, 80, 82

- What patterns do you notice?
- How does the second number compare to the first number?
- How does the third number compare to the first number?

3. Consider the following numbers:

45, 47, 49

103, 105, 107

29, 31, 33

157, 159, 161

- What patterns do you notice?
- How does the second number compare to the first number?
- How does the third number compare to the first number?

Numbers that follow each other in order, without gaps, are called _____.

- Create an expression for if you didn't know the first number, but knew they were consecutive:
 - Pattern in Problem 1: _____
 - Pattern in Problem 2: _____
 - Pattern in Problem 3: _____

 Consecutive Numbers

Consecutive Numbers Chart				
Type of Consecutive Numbers	Examples	Expressions for Terms		
		First	Second	Third
Consecutive Numbers	4, 5, 6 27, 28, 29	x	$x + 1$	$x + 2$
Consecutive Even Numbers	8, 10, 12 62, 64, 66	x	$x + 2$	$x + 4$
Consecutive Odd Numbers	23, 25, 27 89, 91, 93	x	$x + 2$	$x + 4$

1. The sum of three consecutive numbers is 72. What is the smallest of these numbers?

Variables: _____

Equation: _____

2. Find three consecutive odd integers whose sum is 261.

Variables: _____

Equation: _____

Write an equation that can be used to model the following problem. Finally, use your equation to SOLVE the problem.

1. Find three consecutive integers whose sum is 171.

Equation: _____

2. The sum of 3 consecutive even numbers add up to 1002. Find the three numbers.

Equation: _____

3. The sides of a triangular birdcage are consecutive integers. If the perimeter is 114 centimeters, what is the length of each side?

Equation: _____

Day 6 – Isolating a Variable: Simple Equations – Notes

Isolating a variable simply means to solve for that variable or get the variable “by itself” on one side of the equal sign (usually on the left). Sometimes we may have more than one variable in our equations; these type of equations are called **literal equations**. We solve literal equations the same way we solve “regular” equations.

Steps for Isolating Variables

1. Locate the variable you are trying to isolate.
2. Follow the rules for solving equations to get that variable by itself.

Solving an Equation You're Familiar with	Solving a Literal Equation
$2x = 10$	$gh = m$ solve for h
$2x + 5 = 11$	$ax + b = c$ solve for x

Practice:

1. Solve the equation for b: $a = bh$
2. Solve the equation for b: $y = mx + b$

3. Solve the equation for x: $2x + 4y = 10$
4. Solve the equation for m: $y = mx + b$

5. Solve the equation for w: $p = 2l + 2w$

6. Solve the equation for a: $\frac{a}{2} - 1 = b$

Your Turn:

7. Solve the equation for y: $6x - 3y = 15$

8. Solve the equation for h: $V = \frac{1}{3}Bh$

9. Solve the equation for C: $C = \frac{5}{9}(F - 32)$

10. Solve the equation for h: $A = \frac{bh}{2}$

Day 6 – Isolating a Variable: Simple Equations – Practice

Rewrite each equation in terms of the indicated (Letter).

1. $E = mc^2$ for (m)

2. $A = \frac{bh}{2}$ for (b)

3. $S = 2\pi rh$ for (h)

4. $m = \frac{2E}{v^2}$ for (E)

5. $6x - 3y = 15$ for (y)

6. $-9x - 3y = 6$ for (y)

7. $P = 2L + 2W$ for (W)

8. $-15x + 5y = -25$ for (y)

9. $V = \frac{1}{3}Bh$ for (h)

10. $V = \pi r^2 h$ for (h)

11. $A = h(b + c)$ for (b)

12. $s = \frac{w - 10e}{m}$ for (w)

Day 7 – Isolating a Variable: Complex Equations – Notes

One of the most important skills you will encounter for the next two units is the ability to take an equation in standard form ($Ax + By = C$) and solve for y . You had a few problems from yesterday like this, but take some time to practice a few more.

a. $5x - 2y = 8$

b. $-3x + 3y = 6$

c. $-7x - 4y = 12$

Complex Literal Equations

a. $\frac{5x+y}{a} = 2$ for y

b. $c = \frac{3}{4}y + b$ for y

c. $P = \frac{1.2W}{H^2}$ for W

d. $p(t + 1) = -2$, for t

e. $\frac{3ax-n}{5} - 4$ for x

f. $\frac{34-A}{2} = H$ for A