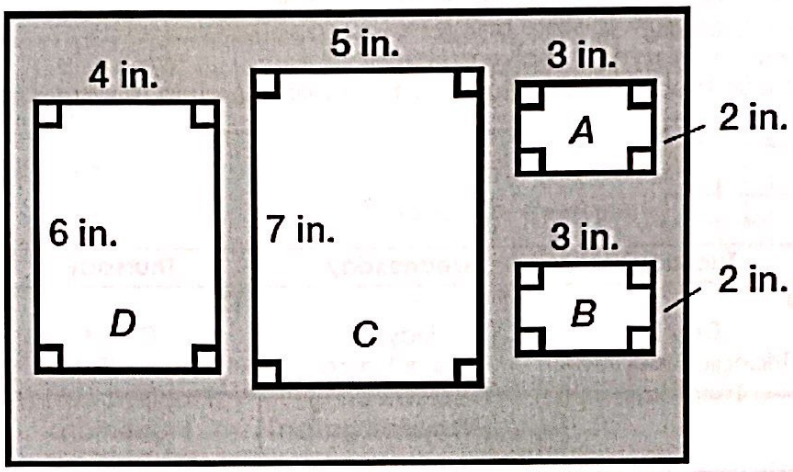
Day 1 – Similarity and Scale Factors – Notes

Scenario: You are creating your own collage of pictures. You bought a large frame and will cut out rectangular holes in the mat as shown.



1. What are the interior angle measures of each mat opening?

90°

2. Write a ratio that compares the length of rectangle A to the length of rectangle B. Then write a ratio that compares the width rectangle A to the width of rectangle B. What do you notice? (Let length be the longer side and width be the shorter side)

$\frac{\text{Rect A}}{\text{Rect B}}$ (length) $= \frac{3}{3} = 1$ $\quad \frac{\text{Rect A}}{\text{Rect B}}$ (width) $= \frac{2}{2} = 1$

They are the same.

3. Write a ratio that compares the length of rectangle A to the length of rectangle D. Then write a ratio that compares the width of rectangle A to the width of rectangle D. What do you notice?

$\frac{\text{Rect A}}{\text{Rect D}}$ (length) $= \frac{3}{6} = \frac{1}{2}$ $\quad \frac{\text{Rect A}}{\text{Rect D}}$ (width) $= \frac{2}{4} = \frac{1}{2}$

They are the same.

4. Write a ratio that compares the length of rectangle A to the length of rectangle C. Then write a ratio that compares the width of rectangle A to the width of rectangle C. What do you notice?

$\frac{\text{Rect A}}{\text{Rect C}}$ (length) $= \frac{3}{7}$ $\quad \frac{\text{Rect A}}{\text{Rect C}}$ (width) $= \frac{2}{5}$

They are different

Two figures are **similar** when the corresponding angles are congruent and the ratios of the measures of the corresponding sides are equal. If two figures are congruent, they are also similar. If two figures are similar, they do not have to be congruent.

Which rectangles are similar?

A and D

B and D

Which rectangles are congruent?

A and B

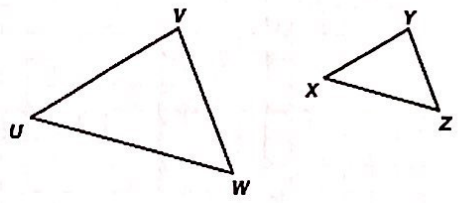
Which rectangles are neither?

A and C

B and C

D and C

When you say two figures are similar to each other, you use the symbol ' ~ '. In the figure below, $\Delta UVW \sim \Delta XYZ$. The order in which you write the vertices in a similarity statement indicates the corresponding angles and sides (just like congruence statements). Name the corresponding sides and angles.



$\angle U \cong \angle X$
$\angle V \cong \angle Y$
$\angle W \cong \angle Z$

$\overline{UV} \sim \overline{XY}$
$\overline{VW} \sim \overline{YZ}$
$\overline{UW} \sim \overline{XZ}$

$\frac{UV}{XY} = \frac{VW}{YZ} = \frac{UW}{XZ}$

(sides are proportional
↳ ratios are =

A **dilation** is a proportional enlargement or reduction of a figure through a point called the **center of dilation**. The size of the enlargement or reduction is called the **scale factor** of the dilation.

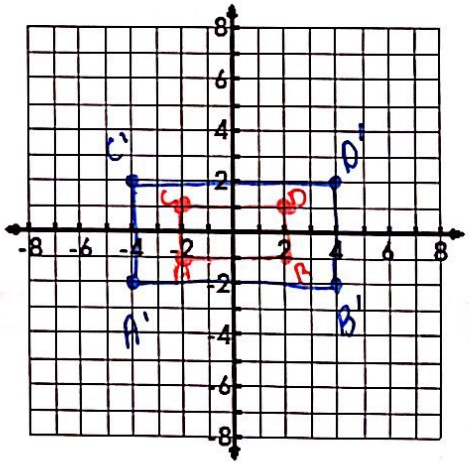
- If the dilated image is larger than the original figure, then the scale factor is greater than 1. This is called an **enlargement.**
- If the dilated image is the same as the original, then the scale factor is 1. The figures are **congruent.**
- If the dilated image is smaller than the original figure, then the scale factor is less than 1. This is called a **reduction.**

A figure and its dilated image are always **similar.** Similar figures will always have the same angle measures, but their side lengths will be different (will remain proportional to each other). **This means dilations do not preserve congruency.** If two figures are congruent, they are also similar.

Example 1: Use a scale factor of 2. Complete the table below and graph both the original (pre-image) and new (image) rectangle.

Pre-Image	Image
A (-2, -1)	A' (-4, -2)
B (2, -1)	B' (4, -2)
C (-2, 1)	C' (-4, 2)
D (2, 1)	D' (4, 2)

Length Ratios
$\frac{8}{4} = 2$
Width Ratios
$\frac{8}{4} = 2$



How did the following change?

A. Angle Measures:

Angle measures stayed the same

B. Length of Sides:

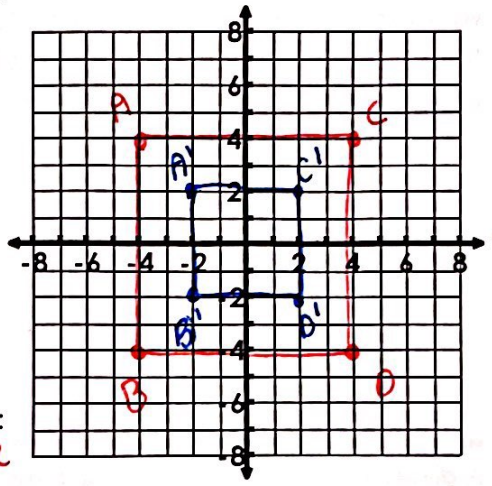
It doubled in size.

Example 2: Use a scale factor of 1/2. Complete the table below and graph both the original (pre-image) and new (image) rectangle.

Pre-Image	Image
A (-4, 4)	A' (-2, 2)
B (-4, -4)	B' (-2, -2)
C (4, 4)	C' (2, 2)
D (4, -4)	D' (2, -2)

Length Ratios
$\frac{8}{4} = 2$

Width Ratios
$\frac{8}{4} = 2$



How did the following change?

A. Angle Measures:
They remained the same.

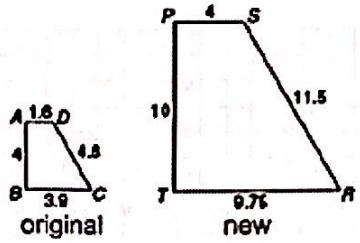
B. Length of the sides:
The lengths were cut in half.

Finding Scale Factors

To find the scale factor of your new figure (image), you want to compare the ratio of the sides from the new figure to the original figure (pre-image).

$$\frac{\text{image}}{\text{pre-image}} = \frac{\text{new}}{\text{original}}$$

Example 3: Trapezoid PTRS is a dilation of Trapezoid ABCD. What is the scale factor of the dilation?

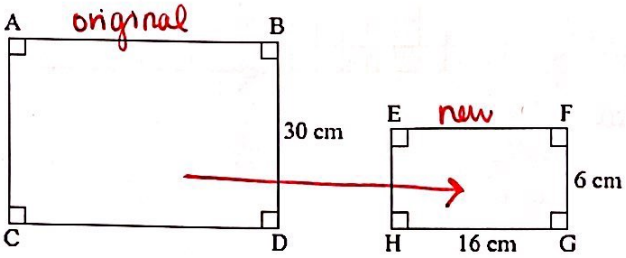


$\frac{4}{1.6} = 2.5$ enlargement

$\frac{1.6}{4} = 0.4$ reduction

The scale factor of the dilation is 2.5.

Example 4: Rectangle EFGH is a dilation of Rectangle ABCD. What is the scale factor of the dilation?



$\frac{30}{6} = 5$ enlargement

$\frac{6}{30} = 0.2$ reduction

Can you find the length of AB?

$16 \times 5 = 80$

AB = 80 cm